Geometric Error Budget Analysis for TerraSAR-X

Othmar Frey[*], Erich Meier[*], Daniel Nüesch[*], Achim Roth[**]
[*]Remote Sensing Laboratories, University of Zurich, Switzerland
email: ofrey@geo.unizh.ch
[**]German Aerospace Center (DLR), Germany

Abstract

The impact of potential error sources on geocoded products has been investigated with respect to the high resolution capabilities of the TerraSAR-X sensor. Datum shift parameters, maps, digital terrain and surface models have been identified as external error sources. The accuracy of the geocoded products depends heavily on the quality and availability of this information, which underlies regional variations. Error sources closely related to the sensor are its position, sampling window start time and Doppler centroid frequency. Another error source is given by atmospheric refraction. Ionospheric and atmospheric path delays have a considerable impact. Appropriate modeling can mitigate this effect. Further, high requirements on radiometric accuracy ask for an improved antenna gain pattern correction, which depends on the actual elevation angle and the terrain height.

1 Introduction

The space-borne synthetic aperture radar system TerraSAR-X is intended to produce high resolution radar images for a broad range of applications and purposes. TerraSAR-X is designed to operate in three different modes, stripmap, ScanSAR and spotlight mode, where the latter should provide image data suitable for image products of up to one meter resolution and a pixel localization accuracy of a couple of meters. Improved system technology imposes stricter requirements on processing and post-processing procedures. A study [3] has been carried out, which focuses on post-processing, namely forward and backward geocoding. The main results of this study are presented in this paper. In a first step, the magnitude of potential errors originating from inaccuracies in orbit position, datum shift parameters, cartography, geoid models, digital terrain models (DTM) and digital surface models (DSM), sampling window start time, Doppler frequency as well as the influence of atmospheric refraction were investigated. Then, individual error budgets were calculated for all these input parameters by simulating forward geocoding as well as backward geocoding based on range-Doppler equations. In addition, the impact of terrain elevation differences within a SAR scene on the radiometric distortion has been investigated in terms of an optimized antenna gain pattern correction.

2 Geocoding Simulation Model

The geocoding simulation is based on the following simplifications and assumptions:

- The earth's surface is modeled by the WGS84 ellipsoid. Earth rotation is described by a constant rotation vector $\vec{\omega}$ with respect to an inertial space.
- All calculations have been performed for zero Doppler centroid frequency.
- The phase center of the antenna is assumed to be equal to the center of mass of the satellite.
- Sending and receiving time are identical.
- Ray bending is neglected.
- The calculations are performed in vacuum space. Atmospheric refraction is considered in the form of an error source, though.
- The satellite orbit is a stationary Keplerian orbit.

Both, forward and backward geocoding, are based on the range-Doppler equations, an orbit propagator based on the nominal Keplerian elements and an ellipsoid equation representing the earth.

1. Doppler equation: $f_d = \frac{2\cdot\left(\vec{V}_S-(\vec{\omega}\times\vec{P})\right)\left(\vec{P}-\vec{S}\right)}{\lambda\cdot R}$
2. Range equation: $R^2 = (\vec{S}-\vec{P})(\vec{S}-\vec{P})$
3. Ellipsoid equation: $\frac{P_x^2}{(a+h)^2} + \frac{P_y^2}{(a+h)^2} + \frac{P_z^2}{(b+h)^2} = 1$

   where: f_d: Doppler centroid frequency, $\vec{P}$: target point, $\vec{S}$: sat. position, $\vec{V}_S$: sat. velocity, $\vec{\omega}$: earth rotation, λ: radar wavelength, R: range, a, b: semi major/minor axis, h: height above ellipsoid.

This set of non-linear equations is solved iteratively. To provide a means to analyse the geocoding behavior in non-flat topography, an inclined plane is used instead of the ellipsoid model. The computation of an error budget contribution resulting from a specific input error works as follows: First, a correct geocoding is carried out for a certain range-Doppler position without having any errors introduced. Then, an erroneous geocoding is calculated using input parameters deteriorated by a selected input error. The difference in ground position (forward geocoding) or image position (backward geocoding), respectively, yields the resulting error.

3 Error Parameters

This section lists the various error sources that were identified. In addition, possible magnitudes of errors are given.

Orbit According to [2] three different classes of orbit accuracies will be supplied:

GPS restituted	10 m rms
GPS precise	2 m rms
GPS high-precise	0.1 m rms

Geometric Error Sources To this category belong errors in datum shift parameters between local and global reference systems, map errors (systematic net distortions, random errors due to generalization, production process etc.), inaccuracies in geoid models, digital terrain and surface models. The magnitude of these errors depends heavily on the availability and quality of regional geodetic and cartographic information. In our study, disposable reference values of Western European countries were used. For a detailed listing we refer to [3]. Each of these error sources must be expected to contribute an error of one to several meters. Some reference values are given in the results section.

Atmospheric Refraction The path delay due to ionospheric and tropospheric refraction leads to a range error. The difference in sensor position at reception time due to the signal delay is neglected in the simulation.

Ionospheric path delay can be modeled as a function of the total electron content TEC and the frequency of the transmitted signal. The TEC is the number of electrons in a column of one meter-squared cross-section along a path through the ionosphere. According to [6] the one way zenithal ionospheric path delay ΔR^Z_{iono} can be modeled as:

$$\Delta R^Z_{iono} = \frac{40.3 \cdot TEC}{f^2} ,$$

where: ΔR^Z_{iono}: one way ionospheric zenithal path delay, f: carrier frequency $[Hz]$, TEC: total electron content $[TECU]$ = $[10^{16}\ \frac{electrons}{m^2}]$. As an approximation, the one way path delay ΔR^{θ}_{iono} at off-nadir angle θ can be modeled by the one way zenithal path delay divided by $\cos(\theta)$: In worst cases, i.e. at far range ($\theta = 40.9^o$) and a $TEC = 150\ TECU$ and more, a delay of close to 1 m results from ionospheric refraction.

Tropospheric refraction results in a one way total zenithal path delay of $ZPD \approx 2.3 - 2.6\ m$ [1] consisting of $2.3\ m$ hydrostatic path delay and $0 - 0.3\ m$ wet path delay. As the troposphere is the lowest of the atmospheric layers touching the earth's surface the tropospheric path delay depends on the target's height above sea level. A maximal one way path delay of ca. $3.3\ m$ is reached at far range (off-nadir angle $\theta = 40.9^o$) and sea level.

Doppler Centroid Frequency Error and Sampling Window Start Time Error Having no precise specifications how accurately the Doppler centroid and the sampling window start time will be known values were chosen, which provoke target position errors that are critical for the accuracy requirements. Doppler centroid errors from $1\ Hz$ to $10\ Hz$ and sampling windows start time errors equivalent to $1\ m$ - $10\ m$ were evaluated.

4 TerraSAR-X Accuracy Requirements

The pixel localization accuracy for spotlight mode using ground control points and an ideal (i.e. error-free) DTM or using no DTM, respectively, is estimated in [5] as:

off-nadir $\theta = 18.5^o$	$8.7\ m$	(without DTM)
off-nadir $\theta = 40.9^o$	$4.2\ m$	
off-nadir $\theta = 18.5^o$	$0.6\ m$	(with ideal DTM)
off-nadir $\theta = 40.9^o$	$0.3\ m$	

For ScanSAR mode, the pixel localization accuracy is required to be $\leq 8.5\ m$ according to [5].

5 Results of Error Budget Analysis

Orbit To fulfill the requirements of spotlight mode a highly precise two-band GPS solution, providing an accuracy of $0.1\ m$ rms is desirable. ScanSAR mode allows the precise orbit determination mode (2 m rms). The lowest orbit accuracy level (10 m rms) is only applicable for products of lower accuracy level than specified in the requirements above.

Datum Shift, Maps, Geoid, Digital Terrain and Surface Models As long as one global reference system – e.g WGS84 or ITRF – is used for internal and auxiliary data, no datum shift is needed. Hence, no errors are introduced from this source. However, auxiliary data is mostly based on local reference frames, in practice. In case of forward geocoding, the datum shift errors are fully passed on to the positioning accuracy leading to errors greater than $10\ m$, unless new reference frames as ITRF and their local equivalents are used. In the latter case, errors can be reduced to $cm - dm$ level.

For high accuracy products based on spotlight mode data backward geocoding is applied using a digital terrain model and ground control points. With the help of ground control points it is possible to convert the data to the local reference frame, balancing the datum shift errors and random errors from maps to a certain extent. What remains are systematic map errors due to geodetic net distortion, height errors due to inaccurate height datums and errors in the digital terrain models. Auxiliary data should be based on maps of scale $1 : 25000$, at least. The quality of maps, DTMs and national geodetic networks are key matters to achieve a highly accurate geocoding.

Using a DTM of good quality – as e.g. the Swiss DHM25 level 2, which features a grid spacing of $25\ m$ and additional break lines, has an average error of $1.5\ m$ for relatively flat areas and an average error of $3\ m$ for alpine regions – geocoding errors of several meters introduced by the DTM must be expected for discontinuous topography.

Coordinates of ground control points extracted from good maps of scale $1 : 25000$ may have a systematic error of ca. $1\ m$. Random errors due to generalization, map production process and map reading sum up to more than $10\ m$. However, these random errors are balanced to a certain extent when using a large set of ground control points.

Transformations between ellipsoidal and orthometric heights need a geoid model. Local geoid models of good quality may provide accuracies of $cm - dm$ level whereas continental and global models have errors of $1 - 5\ m$.

In addition, depending on vegetation and size of man-made objects covering the earth, differences between digital terrain models and the surface, which the sensor can "see", become huge and introduce major image errors – an image error of close to $30\ m$ in the range coordinate for a height error of $30\ m$, for instance.

Sampling Window Start Time Sampling window start time (SWST) errors were chosen to be critical for spotlight mode in this study, because no a priori information was available. Critical values were chosen to visualize potential effects. In case where no ground control points are used a range error of $10\ m$ due to a sampling window start time error may lead to errors of $\sim 30\ m$ on the ground, a range error of $1\ m$ to an error of $\sim 3\ m$, respectively.

Atmospheric Path Delay The impact of ionosphere and troposphere on signal propagation leads to maximal errors of close to $4\ m$ in the image if these effects are not modeled. Simple modeling without needing auxiliary data allows a significant reduction of the range error to $1\ m$ and below, however. The models that were used so far are very basic.

Doppler Centroid Frequency As for the SWST the accuracy of the Doppler centroid frequency was not specified a priori. To show the relationship values that are critical for the spotlight mode requirements were chosen: A Doppler centroid error of $1\ Hz$ leads to a maximal error of $1.4\ m$ in case of forward geocoding. For backward geocoding the error is $1.6\ m$ at the worst. This does not satisfy the spotlight mode requirements. A Doppler centroid error of $10\ Hz$ would become critical even for ScanSAR mode.

Combination of Errors In this study, all error sources have been looked at, individually. Many of them may occur simultaneously and, in the worst case add up linearly to a much bigger error, though. A look at the results reveals that most maximal errors are critical for spotlight mode even if they occur individually. Combinations of various error sources, which reflects the reality, are likely to lead to errors, which exceed spotlight mode requirements. As stated before, errors introduced by datum shift, maps and geoid models may be eliminated to a certain extent using ground control points. To estimate the total error, detailed information about the quality and availability of auxiliary data that will be used for geocoding is needed for individual situations.

6 Elevation Antenna Gain Pattern Analysis

The requirements for the relative radiometric accuracy as specified in [5] are: $0.37\ dB$ for near range (off-nadir $\theta = 18.5^o$) and $0.38\ dB$ for far range (off-nadir $\theta = 40.9^o$). The relative radiometric accuracy defines the difference in measured cross section of equivalent targets measured at the same time at different locations within the product coverage.The absolute radiometric accuracy is $0.65\ dB$ for all off-nadir angles.

A common way to perform the antenna gain pattern correction is to model the elevation angle of each backscatterer as a slant-range dependent function. This approximation holds only in case of very flat terrain. Actually, antenna gain pattern and backscatterer are connected via the elevation angle. In mountainous areas, situations occur, where two backscatterers have the same range distance, although their elevation angles differ completely. A stringent antenna gain pattern correction requires knowledge about the geometric constellation between each backscatterer and the sensor. With the help of a digital terrain model the correct elevation angle information for each pixel can be provided.

To show possible radiometric errors that occur when using the range dependent approach, elevation difference angles and their corresponding radiometric errors in dB have been calculated for backscatterers, which have different heights above sea level but the same range distance. We distinguish between *near range* and *far range* off-nadir angles (near range := off-nadir angle $\theta = 18.5^o$, far range := $\theta = 40.9^o$ – as well as *center of beam* and *edge of beam* constellations. *Center of beam* means that the two backscatterers, which have the same range distance but differing height and horizontal positions, each lie symmetrically on either side of the beam center by half the elevation difference angle. *Edge of beam* means that one of the backscatterers lies at the $-3\ dB$ margin (i.e. at the edge region of the beam cone, which is cut at $-3\ dB$) and the second backscatterer lies closer to the beam center by the amount of the elevation difference angle. For the simulation, the nominal two-way elevation antenna gain pattern was used. **Figure 1** shows the radiometric variation for a set of height differences in the case where a range dependent correction approach is used instead of an elevation angle and terrain dependent one.

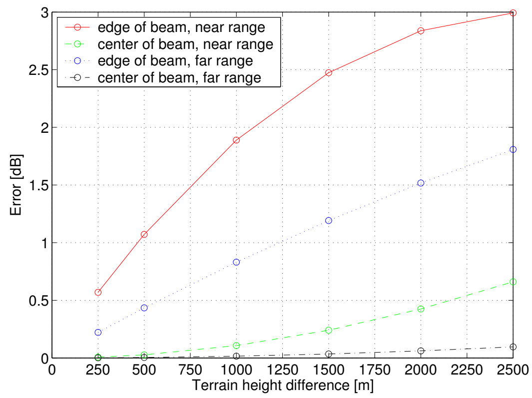

Figure 1: Radiometric errors for various height differences based on the nominal elevation antenna gain pattern.

7 Conclusions

Geometric Errors Automatic geocoding of spotlight mode data is subject to some limitations: Actual pixel localization errors for spotlight mode data may exceed the accuracy requirements. The accuracy is mainly restricted by the quality of digital terrain and surface models. DTMs of sufficient quality are not available on a global scale, yet. Vegetation-covered and built-up areas degrade the accuracy – available DSMs (e.g. from SRTM) might be not accurate enough.
In addition, the quality of maps required for extraction of ground control points is not met for many areas in the world and global geoid models are of insufficient quality. Degradations due to datum shift errors heavily depend on the availability of local frames connected to the new, dynamic reference frames. A dual-band GPS solution providing highly accurate orbit determination is desirable for premium quality geocoding of spotlight mode data. Atmospheric refraction must be considered. Finding an appropriate modeling approach requires further research.

Antenna Gain Pattern Correction The radiometric accuracy requirements are not met even for rather moderate topography if a range dependent approach is used. A height difference of $250\ m$ produces a radiometric error of $0.6\ dB$ at an off-nadir angle $\theta = 18.5^o$ in the worst case, i.e. at the edge region of the radar beam. For a height difference of $500\ m$, the error already considerably exceeds the requirements for the absolute radiometric accuracy. Therefore, a stringent, elevation angle dependent correction approach seems to be inevitable.

Resulting Work As a result of the presented study an in-depth investigation into the appropriate atmospheric models is being carried out [4] and will be implemented in the geocoding software at DLR. A radiometric correction approach incorporating terrain variation and the elevation antenna gain pattern is being implemented [7].

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